Solution 7

Exercise 1:

a) For a 2 terminal device, the two cells are connected in series. The current of the total device is always limited by the sub-cell with the smallest current. In this case the a-Si:H cell current ($16.8\,\text{mA/cm}^2$) will limit the current of the c-Si subcell. Therefore the c-Si will only have an efficiency of

$$\text{Eff}_{c-Si} = 25\% \cdot \frac{16.8\,\text{mA/cm}^2}{41.8\,\text{mA/cm}^2} = 10\%$$

Hence in the (not realistic) best case, neglegting light absorption in the top a-Si cell, we get an efficiency of: $\text{Eff}_{tot} = 10\% + 10\% = 20\%$. This is lower than the c-Si cell's efficiency alone.

b) For a 4 terminal device no more current matching is needed. For simplification we assume that there is no parasitic absorption in the insulating layer that separates the two cells. As a first approximation, the light absorbed in the a-Si:H cell cannot be reused in the c-Si cells. Therefore the efficiency of the c-Si will be reduced by the ratio of the Jsc. The efficiency of the c-Si cells will be:

$$\text{Eff}_{c-Si} = 25\% \cdot \left(1 - \frac{16.8\,\text{mA/cm}^2}{41.8\,\text{mA/cm}^2}\right) = 15.0\%$$

Adding the efficiency of the a-Si cell we get a total efficency of $\text{Eff.}_{tot} = 15.0\% + 10\% = 25.0\%$. So, even in the irrealistic best case the tandem device is not better than the c-Si cell alone. The lower FF of the a-Si cell compared to the c-Si cell cannot be compensated by the better use of higher energy photons of the a-Si cell. (a-Si has a higher band gap which is implied in the higher Voc.)

c) Comparing to a c-Si cell the microcrystalline cells have a lower FF. Therefore it is worth to add an a-Si cell that exploit better the high energy photons.

Exercise 2:

a) The absorption coefficient of silicon increases with photon energy. Therefore, it makes sense to use the cell with a larger bandgap - which is sensitive in the blue range - as the top cell, which lets pass enough red light to be absorbed in the bottom cell. From the rule of thumb $V_{\text{oc}} \approx \frac{2}{3}E_{\text{g}}$ it is clear, that cell a has a smaller bandgap than cell **b**. Therefore, it is used as the bottom cell and cell b as the top cell.

b) To interconnect the cells b and a monolithically means to connect them in series. Therefore, for each current the corresponding voltages are added resulting in the following I-V curve:

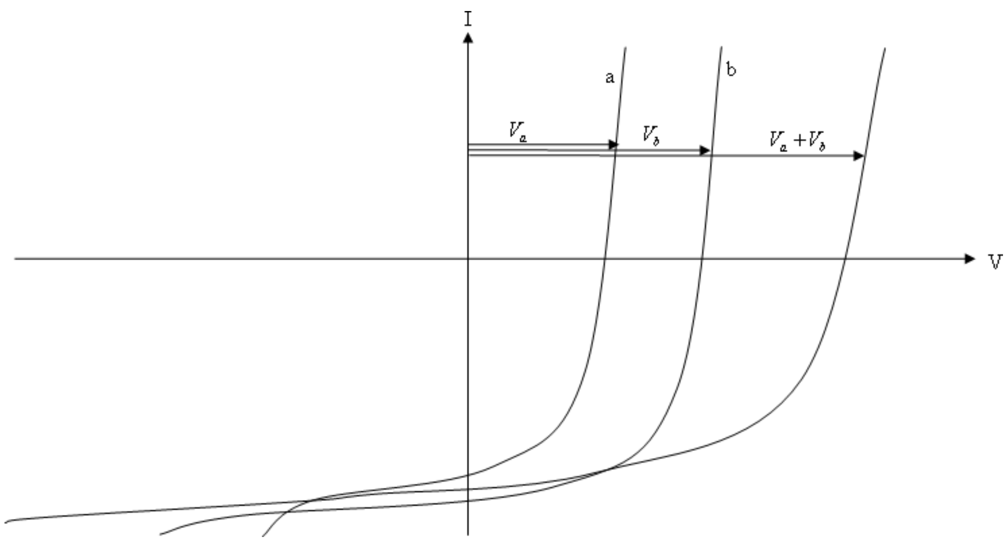

Figure 1: I-V curves of two solar cells a and b and the curve of the monolithically interconnected cell ba.

c) The current at the maximum power point (MPP) is relevant for the question, which cell is the limiting one. From the plot one can see that $I^{\mathrm{a}}_{\mathrm{MPP}} < I^{\mathrm{b}}_{\mathrm{MPP}}$, therefore cell a is the limiting cell and the tandem cell [ba] is bottom-limited.

Exercise 3: The given charge distribution is justified by the fact that the global charge must remain 0 in the cell. We assume that the (spatial) density N_{db} of the dangling bonds is constant but that their charge state can change within the i-layer. Remember that the dangling bonds are so-called amphoteric states and can acquire positive charge (D^+), negative charge (D^-) or remain neutral (D^0). In our case, we restrict ourselves to D^+ and D^- states. In the following, a step-by-step solving of the Poisson equation for the following (symmetrical) charge distribution within the i-layer is given:

$$\rho(x) = \begin{cases} +qN_{\text{db}} & \text{for x} \in [-d/2; 0] \\ -qN_{\text{db}} & \text{for x} \in \,]0; d/2]\,. \end{cases} \tag{1}$$

a) The 1D Poisson equation for this problem is:

$$\frac{\partial^2 \phi(x)}{\partial x^2} = \begin{cases} \frac{+qN_{\text{db}}}{\epsilon_0 \epsilon_r} & \text{for x} \in [-d/2; 0] \\ \frac{-qN_{\text{db}}}{\epsilon_0 \epsilon_r} & \text{for x} \in \,]0; d/2]\,. \end{cases} \tag{2}$$

By integrating once, the electric field is then given by

$$E(x) = -\frac{\partial \phi(x)}{\partial x} = \begin{cases} -\frac{qN_{\text{db}}}{\epsilon_0 \epsilon_r} x - c_+ & \text{for x} \in [-d/2; 0] \\ \frac{qN_{\text{db}}}{\epsilon_0 \epsilon_r} x - c_- & \text{for x} \in \,]0; d/2]\,. \end{cases} \tag{3}$$

We require here that the electric field has to be continuous at 0. Thus, $c_+ = c_- \equiv C$. By integrating once again, the electric potential is obtained:

$$\phi(x) = \begin{cases} \frac{qN_{\text{db}}}{2\epsilon_0 \epsilon_r} x^2 + Cx + d_+ & \text{for x} \in [-d/2; 0] \\ -\frac{qN_{\text{db}}}{2\epsilon_0 \epsilon_r} x^2 + Cx + d_- & \text{for x} \in \,]0; d/2]\,. \end{cases} \tag{4}$$

For the potential, the following boundary conditions are required:

- Continuity at 0:
$$d_+ = \phi(0^-) \overset{!}{=} \phi(0^+) = d_- \equiv D \tag{5}$$

- $\phi(\frac{d}{2}) \overset{!}{=} 0$:
$$\Rightarrow 0 \overset{!}{=} -\frac{qN_{\text{db}} d^2}{8\epsilon_0 \epsilon_r} + C\frac{d}{2} + D.$$

  From this we get
$$D = \frac{qN_{\text{db}} d^2}{8\epsilon_0 \epsilon_r} - C\frac{d}{2}. \tag{6}$$

- $\phi(-\frac{d}{2}) \overset{!}{=} V_{\text{bi}}$:
$$\Rightarrow V_{\text{bi}} \overset{!}{=} \frac{qN_{\text{db}} d^2}{8\epsilon_0 \epsilon_r} - C\frac{d}{2} + D \tag{7}$$
$$\overset{(6)}{=} \frac{qN_{\text{db}} d^2}{4\epsilon_0 \epsilon_r} - Cd. \tag{8}$$

We then obtain:

$$C = \frac{qN_{\mathrm{db}}d}{4\epsilon_0\epsilon_r} - \frac{V_{\mathrm{bi}}}{d} \tag{9}$$

and

$$D \overset{(6)}{=} \frac{V_{\mathrm{bi}}}{2}. \tag{10}$$

Using equations (3) to (10), we finally obtain

$$\boxed{E(x) = -\frac{\partial\phi(x)}{\partial x} = \begin{cases} -\frac{qN_{\mathrm{db}}}{\epsilon_0\epsilon_r}x - \frac{qN_{\mathrm{db}}d}{4\epsilon_0\epsilon_r} + \frac{V_{\mathrm{bi}}}{d} & \text{for x} \in [-d/2;0] \\ \frac{qN_{\mathrm{db}}}{\epsilon_0\epsilon_r}x - \frac{qN_{\mathrm{db}}d}{4\epsilon_0\epsilon_r} + \frac{V_{\mathrm{bi}}}{d} & \text{for x} \in]0;d/2] \end{cases}} \tag{11}$$

for the electric field and

$$\boxed{\phi(x) = \begin{cases} \frac{qN_{\mathrm{db}}}{2\epsilon_0\epsilon_r}x^2 + \left(\frac{qN_{\mathrm{db}}d}{4\epsilon_0\epsilon_r} - \frac{V_{\mathrm{bi}}}{d}\right)x + \frac{V_{\mathrm{bi}}}{2} & \text{for x} \in [-d/2;0] \\ -\frac{qN_{\mathrm{db}}}{2\epsilon_0\epsilon_r}x^2 + \left(\frac{qN_{\mathrm{db}}d}{4\epsilon_0\epsilon_r} - \frac{V_{\mathrm{bi}}}{d}\right)x + \frac{V_{\mathrm{bi}}}{2} & \text{for x} \in]0;d/2]\,. \end{cases}} \tag{12}$$

for the electric potential.

b) For a vanishing electric field in the middle of the layer, we use (11) at $x = 0$ and set:

$$0 \overset{!}{=} E(0) = -\frac{qN_{\mathrm{db}}d}{4\epsilon_0\epsilon_r} + \frac{V_{\mathrm{bi}}}{d}$$

leading to

$$N_{\mathrm{db}} = \frac{4\epsilon_0\epsilon_r V_{\mathrm{bi}}}{qd^2} = \begin{cases} 7.75\times10^{16}\,\mathrm{cm}^{-3} & \text{for } d = 200\,\mathrm{nm} \\ 8.61\times10^{15}\,\mathrm{cm}^{-3} & \text{for } d = 600\,\mathrm{nm}. \end{cases}$$

c) A typical value for N_{db} for a good i-layer before degradation is $5\times10^{15}\,\mathrm{cm}^{-3}$. After degradation, this value can increase by one or two orders of magnitude (see simulations results in the lecture). Even though our assumptions for the charge distribution are quite rough, the obtained values are not so far away from the reality.

d) Under illumination, the generation of charge carriers will change the charge distribution, leading to less screening of the electric field in the middle of the layer (remember that the voltage between the i-layer is uniquely given by the doped layers, see lecture).